

# LINEAR MIXED MODELS WITH RANDOM EFFECTS

Introduction and Analysis of a Split – Plot  
Experiment with SAS/STAT® Software

# LINEAR MIXED MODEL

Fixed Effect (**X** matrix)

All possible levels of a factor or only the levels of interest.

Random Effect (**G** matrix)

Random sample from a set of potential factor levels.

Repeated Effect (**R** matrix)

Measurements from the same experimental unit over time.

# RANDOM EFFECTS IN AGRICULTURE EXPERIMENTS

Most common

Reps

Blocks

Years

Locations

Subjects

# RANDOM EFFECTS IN AGRICULTURE EXPERIMENTS

Broad inference – tests for fixed effects, after being adjusted for random effects, apply to the larger set from which the random effect levels were chosen.

Interactions between random and fixed effects are usually random.  
Split – plot error terms are random effects

# RANDOM STATEMENT

List random effects and options

There is no default test for factors in the random statement.

Some factors may have nothing to contribute to the model and have a zero covariance estimate. This can be considered an ad hoc test for a factor's importance.

A model with random factors **rep** and **year** may adequately model the random effects. Adding the interaction term **rep\*year** is often unnecessary.

# RANDOM STATEMENT SYNTAX

## **random rep;**

Simplest way to specify a random effect.

## **random intercept / subject=rep;**

Same as above, but more efficient because the procedure processes random effects by subjects. Some model options require this syntax.

Multiple random statements are allowed.

There are many more options for specifying and using random factors.

# LEAST SQUARES MEANS (LS-MEANS)

The only option for estimating treatment level means for mixed models

Equal arithmetic means when:

- the design is balanced
- there are no missing values
- the model has no covariates

Theoretically better than arithmetic means when an experiment has one or more of these conditions.

Lessen the impact of an unbalanced design and missing data.

# LS-MEAN TESTS

An LS-mean estimate doesn't have a value for **N** associated with it.  
LS-mean differences are determined by individual (pair-wise) t-tests.  
There is no difference measure (hurdle value) like an LSD, MSD or HSD for the tests.

The **adjust=** option is used to modify the tests when comparing many treatment levels, i.e. **adjust=tukey**.

**Contrast, estimate and lsmestimate** are available for making more complex comparisons.



# Zero Covariance Parameter Estimate

Covariance Parameter Estimates	
Cov Parm	Estimate
rep	0
rep*nitrogen	0.1993
Residual	0.6032

SASLOG note:

NOTE: Convergence criteria met.

NOTE: Estimated G matrix is not positive definite.

NOTE: Asymptotic variance matrix of covariance parameter estimates has been found to be singular and a generalized inverse was used. Covariance parameters with zero variance do not contribute to degrees of freedom computed by DDFM=KENWARDROGER2.

# Zero Covariance Parameter Estimate

Translation:

The covariance parameter estimate for **rep** was a negative value. By definition, variances cannot be negative, so the procedure set the estimate to zero. The procedure used an alternate algorithm to minimize the  $-\text{LogLikelihood}$  function due to the zero estimate. It converged and the zero estimate did not affect the results of the analysis of variance.

# LS-MEANS VS ARITHMETIC MEANS

Gender	Associate	Full
Male	130 (12)	136 (2)
Female	118 (4)	120 (5)

Mean	Full	$((136 \times 2) + (120 \times 5)) / 7 = 124.6$
	Associate	$((130 \times 12) + (118 \times 4)) / 16 = 127$
LS-mean	Full	$(136 + 120) / 2 = 128$
	Associate	$(130 + 118) / 2 = 124$

Cai, Weiji. (2010). "Making Comparisons Fair: How LS-Means Unify the Analysis of Linear Models," Proceedings of the SAS Global Forum 2010 Conference, Cary, NC: SAS Institute Inc.

# Questions?

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